Why Cross-Multiply?

Applying Common Core's Mathematical Practices in proportional reasoning

Amber Cline-Rabah

Why Do We Cross-Multiply?

Have you ever asked yourself…

- Why is the predominant strategy for solving proportional relationships cross-multiplication?
- Why does cross-multiplying work?
- When students cross-multiply, what mathematical thinking are they doing?

In Today's Session, We Will…

1. Solve proportional reasoning problems.
2. Examine solution strategies and student misconceptions.
3. Use several sense-making strategies for solving proportional problems.
4. Explain the difference between algorithms and sense-making strategies.
5. Leave with something useful (hopefully! ☺)

So that we CAN…

1. Apply the mathematical practices

MP.1	Make sense of problems and persevere in solving them
MP.2	Reason abstractly and quantitatively
MP.4	Model with mathematics
MP.7	Look for and make use of structure
MP.8	Look for and express regularity in repeated reasoning

We'll know we've got it WHEN…

1. we can identify at least three different strategies for solving proportional reasoning problems.
2. we can explain why each strategy makes sense.
3. we can evaluate each strategy's usefulness in a given situation.
4. we wonder why we ever taught kids to cross-multiply!

Mrs. Cline-Rabah WILL…

1. Facilitate discussion of problems and strategies and ask probing questions.

Share Out

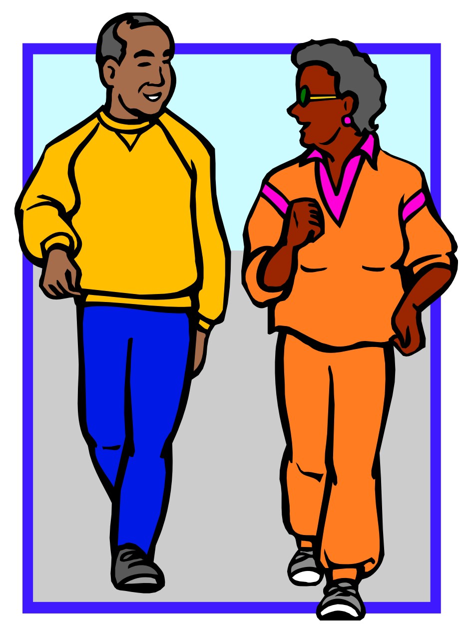
- Why is the predominant strategy for solving proportional relationships cross-multiplication?
- Why does cross-multiplying work?
- When students cross-multiply, what mathematical thinking are they doing?

A Problem to Consider

- A truck traveled 75 miles in 1.5 hours. At the same rate, how long would it take the truck to drive 325 miles?

A Second Problem to Consider

Daniel and Katya are walking partners who walk at the same rate. Since Daniel was running late on Tuesday, Katya started walking without him. By the time Daniel arrived, Katya had already completed three laps. How many laps will Daniel have completed when Katya finishes her tenth lap?



Turn and Talk

- How did you solve each problem?
- How are these questions alike?
- How are they different?
- What kind of reasoning is used in each problem?

Possible Solution Strategies for Proportional Reasoning Problems

Strategy	Description
Build Up	Students use repeated ratios to build up to the unknown quantity. *This strategy only works for multiples of a given ratio.
Scale Factor	Students use a common multiplier to write equivalent ratios.
Relationships within Ratios	Students find relationships within the compared values of a ratio and apply it to an equivalent ratio.
Ratio Tables	Students set up a table and use rate of change to compare the quantities.
Unit Rate	Students identify the unit rate and then use it to solve the problem.
Cross-Multiplication Algorithm	Students set up a proportion (equivalence of two ratios), find the cross products, and solve by using division.

A Classic Proportional Problem

Louise is planting flowers. At the garden store, three bags of potting soil weigh 18 pounds. Knowing that she needed twelve bags, Louise wondered, “How much do twelve bags of potting soil weigh?”

The Build Up Strategy

3 bags weigh 18 lbs

6 bags weigh 36 lbs

9 bags weigh 54 lbs

12 bags weigh 72 lbs

+3 +3 +3

+18 +18 +18

The Scale Factor Strategy

$$\frac{18}{3} = \frac{?}{12} \qquad \cdot 4$$

$? = 72$

What is the scale factor From 3 to12?

Relationships Within Ratios

How are the numerator and denominator of the ratio related to each other?

$$\cdot 6 \quad \frac{18}{3} = \frac{?}{12} \quad \cdot 6$$

$= 72$

Ratio Tables

Bags	Lbs
1	6
2	12
3	18
4	24
5	30
6	36
7	42
8	48
9	54
10	60
11	66
12	72

+3 +3 +3

+18 +18 +18

The Unit Rate Strategy

3 bags = 18 lbs

$$\frac{3 \text{ bags}}{3 \text{ bags}} = \frac{18 \text{ lbs}}{3 \text{ bags}}$$

$$\frac{6 \text{ lbs}}{1 \text{ bag}}$$ unit rate

Then, use one of the previous strategies to find the number of lbs in 12 bags.

Cross-Multiplication Algorithm

- An *algorithm* is a step-by-step procedure for calculations.

- The cross-multiplication algorithm, if set up properly, will result in a correct answer.

- $12 \cdot 18 = 3 \cdot x$
- $216 = 3x$
- $72 = x$

$$\frac{18}{3} = \frac{?}{12}$$

When students cross-multiply are they using proportional reasoning?

- Algorithms are devoid of mathematical meaning when taught directly to students.

- However, when students create algorithms for themselves, they can be meaningful.

Why Cross-Multiplying Works

Make a common denominator, in this case, 36.

$$\frac{18}{3} = \frac{?}{12}$$

$$\frac{18 \bullet 12}{3 \bullet 12} = \frac{? \bullet 3}{12 \bullet 3}$$

Why Cross-Multiplying Works

$$\frac{18 \bullet 12}{3 \bullet 12} = \frac{? \bullet 3}{12 \bullet 3} \rightarrow \frac{216}{36} = \frac{3 \bullet ?}{36}$$

Since the denominators are the same, the numerators are also equivalent. Thus,

$$\frac{216}{3} = \frac{\cancel{3} \bullet ?}{\cancel{3}}$$

$$72 = ?$$

Why Cross-Multiplying Works

In making common denominators, we multiplied $18 \cdot 12$ and $? \cdot 3$, the *cross products*.

To isolate the unknown value, we divided both sides by 3.

Therefore, a quick algorithm for finding the missing piece of a proportion is to cross-multiply and divide by the remaining value.

Memorizing vs. Making Sense

- When we ask students to memorize algorithms, like cross-multiplication, we rob them of the opportunity to make sense for themselves.

- If students can't memorize the algorithm, then they can't do math.

- Students ask, "What was I supposed to do?" instead of "How could I solve this?"

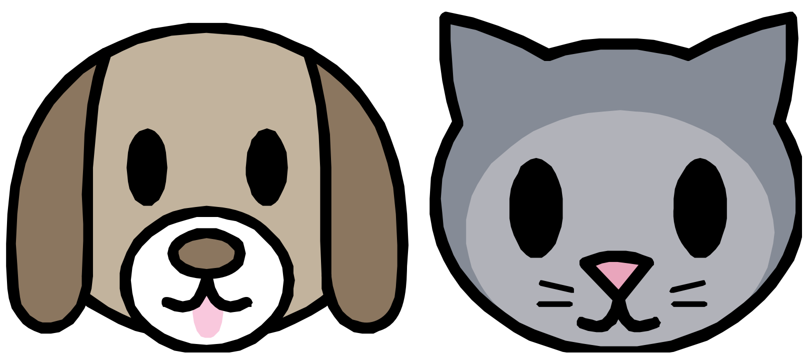
With all of the sense-making strategies available to us,

WHY CROSS-MULTIPLY??

Questions?

Applying What We've Learned

The ratio of cats to dogs at the animal shelter is 8:5. If there are 100 dogs at the shelter, how many cats are there?



Last month there were 650 animals. How many of them were dogs?

One Solution to Dogs & Cats

Which strategy would you employ in this problem?

Which strategy might your students use?

$$\frac{8}{5} = \frac{c}{100} \quad (\cdot 20) \qquad c = 240$$

Part to Whole Reasoning

If the ratio of cats:dogs is 8:3, what is the ratio of dogs:animals?

Part to part and ***part to whole*** reasoning is an essential element of proportional reasoning.

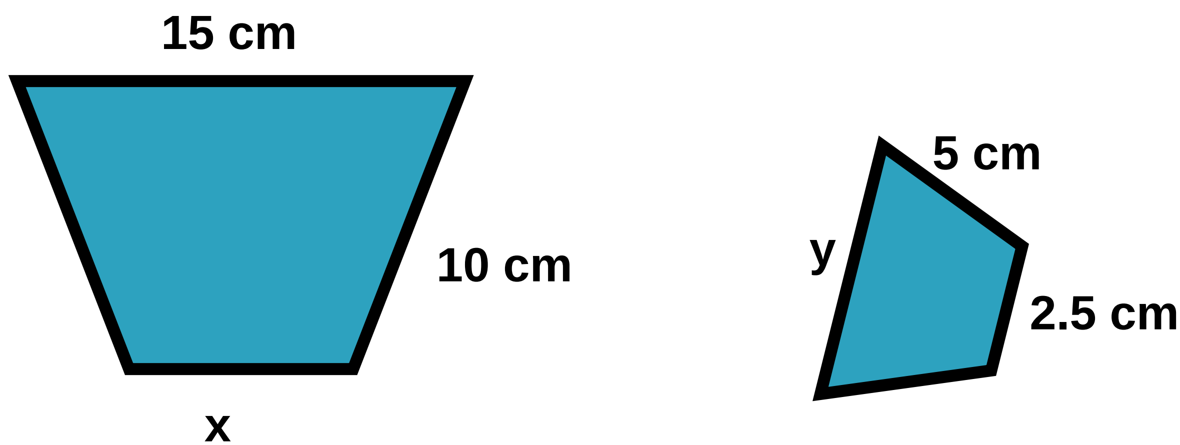
$$\cdot 50$$

$$\frac{5}{13} = \frac{c}{650} = 250$$

$$\cdot 50$$

Applying What We've Learned

The two trapezoids are similar. What is the lengths of the missing sides?



One Solution to Missing Side Problems

Which strategy would you employ in this problem?

Which strategy might your students use?

$= 5$

$$\div 2 \quad \frac{2.5}{5} = \frac{x}{10} \quad \div 2$$

Patterns in Similar Figures

Side	Large Trapezoid	Small Trapezoid
Shortest	x	2.5
Medium	10	5
Longest	15	y

How could students use patterns in a ratio table to find the missing values?

Applying What We've Learned

Eight boys shared three pizzas equally while ten girls shared five pizzas equally. Who ate more pizza –a boy or a girl?

One Solution Strategy

8 boys / 3 pizzas

10 girls / 5 pizzas

What student misconceptions are possible here?

Using unit rates,

$2.\overline{6}$ boys / 1 pizza

2 girls / 1 pizza

So, fewer girls share 1 pizza, thus the girls get more pizza.

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